

# Six-jet production at $e^+e^-$ linear colliders<sup>1</sup>

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## Abstract

The calculation of the tree-level QCD processes  $e^+e^- \rightarrow q\bar{q}gggg$ ,  $q\bar{q}q'\bar{q}'gg$  and  $q\bar{q}q'\bar{q}'q''\bar{q}''$  has recently been accomplished. We highlight here the relevance of such reactions for some of the physics at future electron-positron linear accelerators.

## 1. Motivations for an exact calculation of $e^+e^- \rightarrow 6$ partons

As accelerator physics will enter the Linear Collider (LC) epoch [1, 2], one will encounter a long series of resonant processes ending up with six-jet signatures. One should recall top quark production and decay for a start, whose study will represent one of the main areas of activity at a future LC [3]. Top quarks will be produced in pairs, via  $e^+e^- \rightarrow \gamma^*, Z^* \rightarrow t\bar{t}$ , followed most of the times by  $t\bar{t} \rightarrow b\bar{b}W^+W^- \rightarrow 6$  jets. Then one should not forget the new generation of gauge boson resonances, such as  $e^+e^- \rightarrow ZW^+W^-$  and  $ZZZ$ , and their dominant six-jet decays. The interest in these reactions resides primarily in the possibility of an accurate study of the gauge structure of the electroweak (EW) model [4]. In the same respect, one could also add highly-virtual photonic processes, like  $e^+e^- \rightarrow \gamma^*W^+W^-$ ,  $\gamma^*ZZ$ ,  $\gamma^*\gamma^*Z$  and  $\gamma^*\gamma^*\gamma^*$ , in which the photons split into quark-antiquark pairs. In addition, of particular relevance are reactions involving the Higgs particle,  $\phi$ , e.g., in the Standard Model (SM), such as  $e^+e^- \rightarrow Z\phi \rightarrow ZW^+W^-$  and  $Z\phi \rightarrow ZZZ$  – as discovery channels of a heavy scalar boson [5] – or  $e^+e^- \rightarrow Z\phi \rightarrow Z\phi\phi$  – as a means to study the Higgs potential of a light scalar (the latter decaying to  $b\bar{b}$ ) [6].

Given such a wide scope offered by six-jet final states, it is of paramount importance to have a strong control on the backgrounds. The parton-shower (PS) event generators (e.g., HERWIG [7] and JETSET/PYTHIA [8]) represent a valuable instrument in this respect, as they are able to describe the full event, from the initial hard scattering down to the hadron level. However, Matrix Element (ME) models are acknowledged to describe the large angle distributions of the QCD radiation better than the former do (see, e.g., [9]), which are in fact superior in the small angle dynamics. As in the processes we just mentioned the final state jets are typically produced at large angle and are isolated (being the decay products of massive objects), the need of exact ME computations should be

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manifest<sup>2</sup>. As for theoretical advances in this respect, studies of  $e^+e^- \rightarrow 6\text{-quark EW}$  processes are well under way (see Ref. [11] for a review). However, a large fraction of the six-jet cross section comes from QCD interactions. The case of QCD six-jet production from  $W^+W^-$  decays was considered in Ref. [12]. In this note, we discuss the dominant, tree-level QCD contributions to six-jet final states through the order  $\mathcal{O}(\alpha_s^4)$ , i.e., the processes:

$$e^+e^- \rightarrow \gamma^*, Z^* \rightarrow q\bar{q}gggg, q\bar{q}q'\bar{q}'gg, q\bar{q}q'\bar{q}'q''\bar{q}'', \quad (1)$$

where  $q, q'$  and  $q''$  represent any possible flavours of quarks (massless and/or massive) and  $g$  is a gluon, whose computation has recently been tackled [13].

## 2. Numerical results

In order to select a six-‘jet’ sample we apply a jet clustering algorithm directly to the ‘quarks’ and ‘gluons’ in the final state of the processes (1). For illustrative purposes, we use the Cambridge (C) jet-finder [14, 15] only. This is based on the ‘measure’

$$y_{ij} = \frac{2 \min(E_i^2, E_j^2)(1 - \cos \theta_{ij})}{s_{ee}}, \quad (2)$$

where  $E_i$  and  $E_j$  are the energies and  $\theta_{ij}$  the separation of any pair  $ij$  of particles in the final state, with  $i < j = 2, \dots, 6$ , to be compared against a resolution parameter denoted by  $y$ . In our tree-level approximation, the selected rate is nothing else than the total partonic cross section with a cut  $y_{ij} > y$  on any possible  $ij$  combination. The summations over the three reactions (1) and over all possible ‘massless’ (see Ref. [13] for a dedicated study of mass effects) combinations of quark flavours in each of these have been performed here. As for numerical inputs, they can be found in [13].

The six-jet event rate induced by the  $\mathcal{O}(\alpha_s^4)$  QCD events at  $\sqrt{s_{ee}} = 500$  GeV – the value that we use here for the centre-of-mass (CM) energy of a LC – can be rather large. Adopting a yearly luminosity of, e.g.,  $100 \text{ fb}^{-1}$  and assuming a standard evolution of  $\alpha_s$  with increasing energy, at  $y = 0.001$  one should expect some 2300 events per annum: see Fig. 1. However, these rates decrease rapidly as  $y$  gets larger. From Fig. 1, one can appreciate how the dominant component is due to two-quark-four gluon events, followed by the four-quark-two-gluon and six-quark ones, respectively. These relative rates are of particular relevance to a LC environment. In fact, the capability of the detectors of distinguishing between jets due to quarks (and among these, bottom flavours in particular: e.g., in selecting top and light Higgs decays) and gluons, is of crucial importance there, in order to perform dedicated searches for old and new particles.

The concern about background effects at a LC due to six-jet events via  $\mathcal{O}(\alpha_s^4)$  QCD comes about if one considers that they may naturally survive some of the top signal

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<sup>2</sup>The matching of fixed-order (as well as resummed) multi-parton final states with the subsequent PS to finally reach the hadronisation stage is also a pressing matter, towards which some progress has recently been made [10].

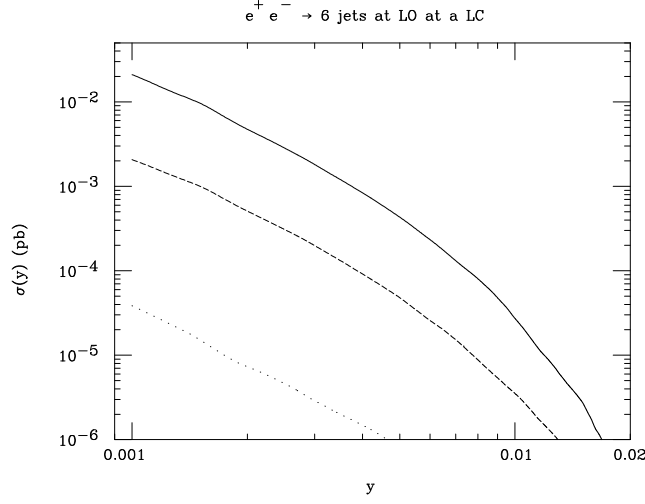


Figure 1: The total cross section of six-jet events at LO in the C scheme, decomposed in terms of the three contributions  $q\bar{q}g g g g$  (solid),  $q\bar{q}q' \bar{q}' g g$  (dashed) and  $q\bar{q}q' \bar{q}' q'' \bar{q}''$  (dotted).

selection criteria. It is well known that the large value of the top mass (here,  $m_t = 175$  GeV) leads to rather spherical events. Therefore, shape variables such as thrust and sphericity represent useful means to disentangle  $e^+e^- \rightarrow t\bar{t}$  events. For example, a selection strategy that does not exploit neither lepton identification nor the tagging of  $b$ -jets was outlined in Ref. [3]. The requirements are a large particle multiplicity, a high

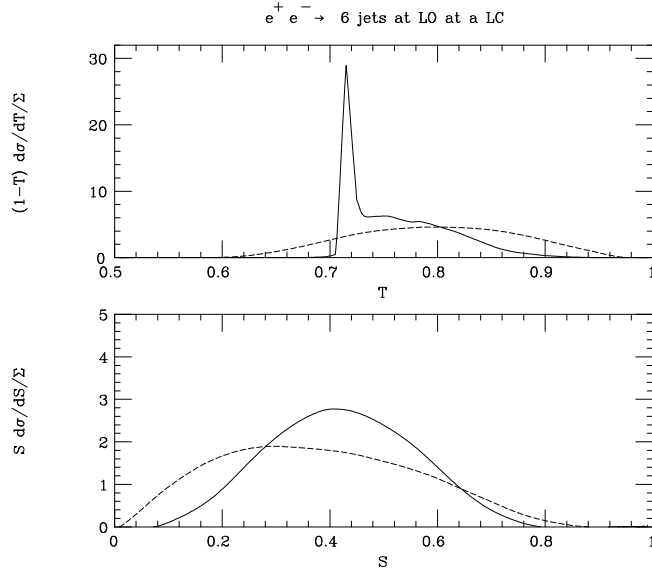


Figure 2: The distributions in thrust (upper plot) and sphericity (lower plot) for  $t\bar{t} \rightarrow 6$ -jet events (solid) and for those of the type (1) (dashed) at LO in the C scheme with  $y = 0.001$ . Notice that the distributions have been normalised to a common factor (one) for readability.

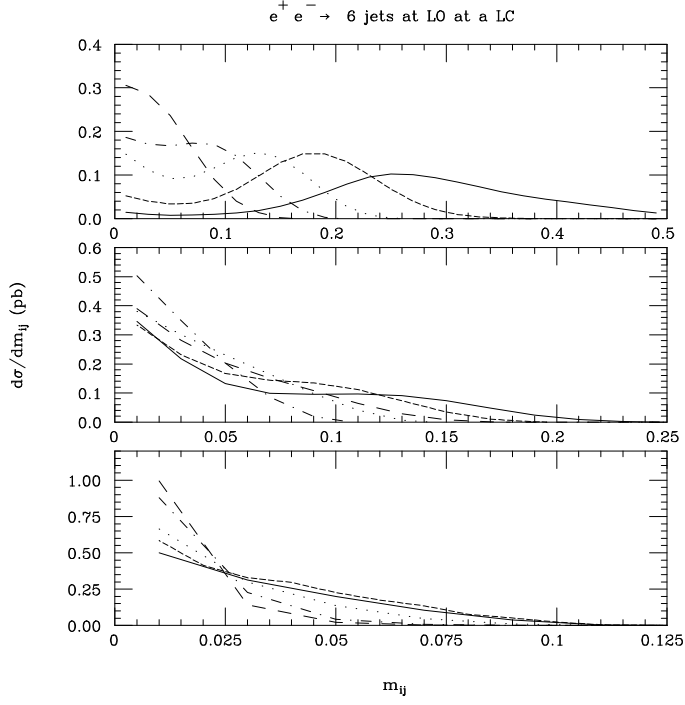


Figure 3: The distributions in the reduced invariant mass  $m_{ij}$  (3) for events of the type (1) at LO in the C scheme with  $y = 0.001$ , for the following combinations of parton pairs  $ij$ : (12)[23]{35} (solid), (13)[24]{36} (short-dashed), (14)[25]{45} (dotted), (15)[26]{46} (dot-dashed) and (16)[34]{56} (long-dashed), in the (upper)[central]{lower} frame.

number of jets (eventually forced to six) and a rather low(high) thrust(sphericity). Jets are selected according to a jet clustering algorithm (in our case, the C one with  $y = 0.001$ , for sake of illustration). Clearly, six-jet events (1) meet the first two criteria. As for the thrust and sphericity distributions, these are shown in Fig. 2. From there, it is evident the overlap of the  $t\bar{t}$  and QCD spectra.

Searches for resonances will often need to rely on the mass reconstruction of di-jet systems. Therefore, it is worth looking at the invariant mass distributions which will be produced by all possible two-parton combinations  $ij$  in (1). As usual in multi-jet analyses, we first order the jets in energy, so that  $E_1 > \dots > E_6$ . Then, we construct

$$m_{ij} \equiv \frac{M_{ij}^2}{s_{ee}} = \frac{2E_i E_j (1 - \cos \theta_{ij})}{s_{ee}}. \quad (3)$$

These fifteen quantities are shown in Fig. 3 for the C scheme at  $y = 0.001$ . We found it convenient to plot the ‘reduced’ invariant masses  $m_{ij}$  rather than the actual ones  $M_{ij}$ , as energies and angles ‘scale’ with the CM energy in such a way that the shape of the distributions is largely unaffected by changes in the value of  $\sqrt{s_{ee}}$  in the CM energy range relevant to a LC. In the figure, it is interesting to notice the ‘resonant’ behaviour of some of the distributions.

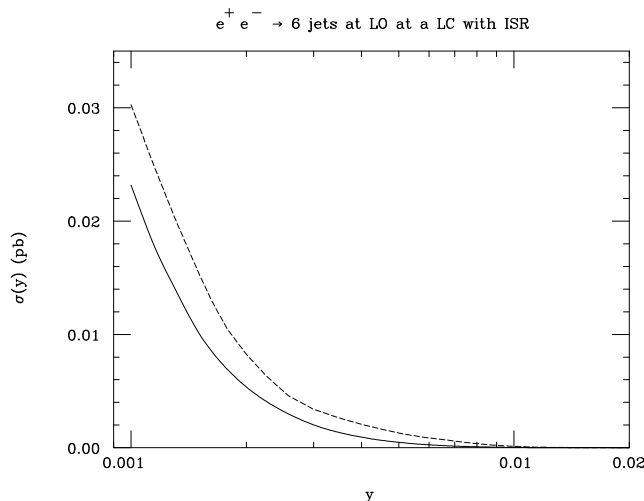


Figure 4: The total cross section of six-jet events at LO in the D scheme, without (solid) and with (dashed) ISR.

It is well known that photon bremsstrahlung generated by the incoming  $e^+e^-$  beams (or Initial State Radiation, ISR) can be quite sizable at a LC<sup>3</sup>. In order to implement ISR we resort to the so-called Electron Structure Function approach by using the  $\mathcal{O}(\alpha_{em}^2)$  expressions of Ref. [18]. As a simple exercise, we plot the production rates for the sum of the processes (1) in presence of ISR in Fig. 4, as a function of the resolution parameter  $y$  in the C scheme. They are compared to those obtained without ISR (that is, the sum of the rates in Fig. 1). We see that the curve corresponding to the processes convoluted with ISR lies above the lowest-order one. This is rather intuitive, as it is well known that the radiation of photons from the incoming electron and positrons tends to lower the ‘effective’ CM energy of the collision [17]. At  $y = 0.001$ , the difference in the rates is around 25% and this tends to increase as  $y$  gets larger. In contrast, we have checked that the shape of the differential distributions (such as those in Fig. 3) suffers little from ISR.

### 3. Summary

The exact calculation of the processes  $e^+e^- \rightarrow q\bar{q}ggggg$ ,  $q\bar{q}q'\bar{q}'gg$  and  $q\bar{q}q'\bar{q}'q''\bar{q}''$  (for both massless and massive quarks) at the leading-order in perturbative QCD has been completed. In this short note we have emphasised the strong impact that such reactions can have as backgrounds to many of the multi-jet studies foreseen at a LC. A numerical program is available for simulations, based on helicity amplitudes, thus also allowing for initial state polarisation.

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<sup>3</sup>Other peculiar feature is the presence of Linac energy spread and beamsstrahlung [16] effects. For a TESLA collider design, these are however smaller in comparison [17], so we leave them aside here.

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